

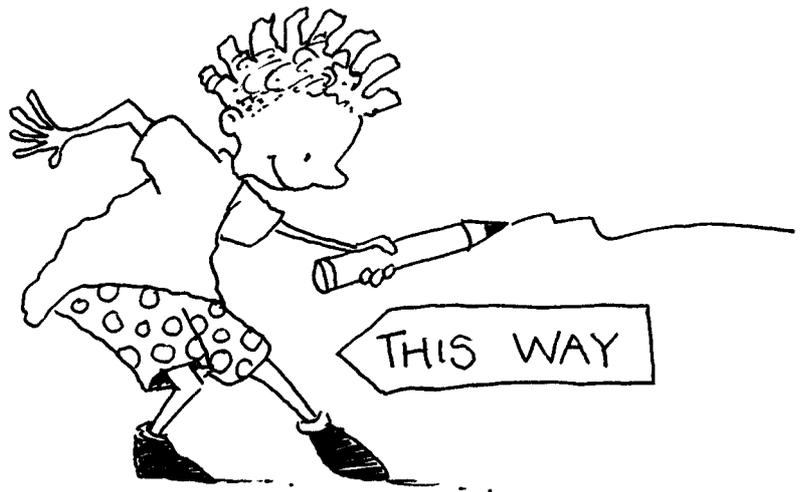
UNIT P5 ■
Problem Solving
Upper Primary



Blake's Topic Bank

Problem Solving Working Backwards

by Sharon Shapiro



This unit contains:

- Teaching notes
- 3 teaching examples
- 1 BLM
- 18 task cards
- Answers

Problem Solving Working Backwards

Sharon Shapiro

Upper Primary



THE PROBLEM SOLVING PROCESS

It is important that students follow a logical and systematic approach to their problem solving. Following these four steps will enable students to tackle problems in a structured and meaningful way.

STEP 1: UNDERSTANDING THE PROBLEM

- ❖ Encourage students to read the problem carefully a number of times until they fully understand what is wanted. They may need to discuss the problem with someone else or rewrite it in their own words.
- ❖ Students should ask internal questions such as, what is the problem asking me to do, what information is relevant and necessary for solving the problem.
- ❖ They should underline any unfamiliar words and find out their meanings.
- ❖ They should select the information they know and decide what is unknown or needs to be discovered. They should see if there is any unnecessary information.
- ❖ A sketch of the problem often helps their understanding.

STEP 2: STUDENTS SHOULD DECIDE ON A STRATEGY OR PLAN

Students should decide how they will solve the problem by thinking about the different strategies that could be used. They could try to make predictions, or guesses, about the problem. Often these guesses result in generalisations which help to solve problems. Students should be discouraged from making wild guesses but they should be encouraged to take risks. They should always think in terms of how this problem relates to other problems that they have solved. They should keep a record of the strategies they have tried so that they don't repeat them.

Some possible strategies include:

- ❖ Drawing a sketch, graph or table.
- ❖ Acting out situations, or using concrete materials.
- ❖ Organising a list.
- ❖ Identifying a pattern and extending it.
- ❖ Guessing and checking.
- ❖ Working backwards.
- ❖ Using simpler numbers to solve the problem, then applying the same methodology to the real problem.
- ❖ Writing a number sentence.
- ❖ Using logic and clues.
- ❖ Breaking the problem into smaller parts.

STEP 3: SOLVING THE PROBLEM

- ❖ Students should write down their ideas as they work so they don't forget how they approached the problem.
- ❖ Their approach should be systematic.
- ❖ If stuck, students should reread the problem and rethink their strategies.
- ❖ Students should be given the opportunity to orally demonstrate or explain how they reached an answer.

STEP 4: REFLECT

- ❖ Students should consider if their answer makes sense and if it has answered what was asked.
- ❖ Students should draw and write down their thinking processes, estimations and approach, as this gives them time to reflect on their practices. When they have an answer they should explain the process to someone else.
- ❖ Students should ask themselves 'what if' to link this problem to another. This will take their exploration to a deeper level and encourage their use of logical thought processes.
- ❖ Students should consider if it is possible to do the problem in a simpler way.

Teaching Notes Working Backwards



The strategy of working backwards is used to solve problems that include a number of linked factors or events, where some of the information has not been provided, usually at the beginning of the problem. To solve these problems it is usually necessary to start with the answer and work methodically backwards to fill in the missing information.

This strategy is extremely useful in dealing with a situation or a sequence of events. The events occur one after the other and each stage, or piece of information, is affected by what comes next. Students begin at the end, with the final action, and work through the process in reverse order to establish what happened in the original situation.

In order to use the strategy of working backwards effectively, students will need to develop the following skills and understanding.

USING THE OPPOSITE OPERATION WHEN WORKING BACKWARDS

When you are solving a problem by starting at the end and working backwards, any mathematical operations you come across will have to be reversed. This means that if the problem requires you to add something, then when working backwards you must subtract it, or if multiplying when working forwards, you must divide when working backwards.

So if the problem the correct way round is –

$$\boxed{?} \div 8 \times 2 = 14$$

then backwards it will be –

$$14 \div 2 \times 8 = 56$$

Or

Jack is 35 years younger than Karen. Frank is half of Jack's age. Jennifer is 17 years older than Frank. If Jennifer is 35 years old, how old is Karen?

Jennifer is 35. She is 17 years older than Frank. So using the opposite operation plus becomes minus. So Frank is $35 - 17 = 18$

Frank is half Jack's age so the opposite operation is $18 \times 2 = 36$

Jack is 35 years younger than Karen so $36 + 35 = 71$

Therefore Karen is 71 years old.

STARTING WITH THE ANSWER AND WORKING BACKWARDS

In a problem where you know the final outcome but don't know the starting point, beginning at the end of the problem and working backwards is the best way of arriving at a solution.

For example, in a dancing competition all the contestants started dancing together. After three minutes half the people were eliminated. During the next ten minutes half of the remaining were eliminated. At the 15 minute mark, half again were eliminated, and at the 20 minute mark, half of those still remaining were eliminated. In the last two minutes one more contestant was eliminated leaving a winner of the competition. How many dancers were there in the beginning?

You know that there is one winner and that the number of contestants was halved at certain intervals. Using this information, it is possible to work backwards and find out how many dancers entered the competition.

Start with the winner		1 person dancing
Last 2 minutes	1+1	= 2 dancers
After 20 minutes	Double	= 4 dancers
After 15 minutes	Double	= 8 dancers
After 10 minutes	Double	= 16 dancers
After 3 minutes	Double	= 32 dancers

32 dancers started.



Teaching Examples Working Backward



EXAMPLE 1

John is four years younger than Carmel but Jane is 24 years older than Carmel. If Jane is 35, how old is John?

Understanding the problem

WHAT DO WE KNOW?

John is four years younger than Carmel.
Jane is 24 years older than Carmel.
Jane is 35 years old.

WHAT DO WE NEED TO FIND OUT?

Questioning:

How old is John?

Planning and communicating a solution

Begin with the information you know, Jane's age, and work backwards to calculate John's age.

Jane is 35 years old. She is 24 years older than Carmel.

So, $35 - 24 \text{ years} = 11$.

Therefore, Carmel is 11 years old.

John is four years younger than Carmel.

so, $11 - 4 = 7$

Therefore, John is seven years old.

Reflecting and generalising

By starting with the known factor of Jane's age we were able to work backwards and calculate the answer. This strategy can be applied to problems which include a sequence of events where we know the end result but don't know the starting point. You can check your answer by working forwards through the problem to see if you reach the correct end point.



Extension

Additional people who are older and younger can be added to further complicate the problem. Students can construct their own problems using the ages of their families or friends.



Teaching Examples Working Backwards



EXAMPLE 2

Four students in the class weighed themselves. Carter was 15 kilograms lighter than Adrian. Gary was twice as heavy as Carter and Jeremy was seven kilograms heavier than Gary. If Jeremy weighed 71 kilograms what was Adrian's weight?

Understanding the problem

WHAT DO WE KNOW?

There are four students in the class.
Carter was 15 kilograms lighter than Adrian.
Gary was twice as heavy as Carter.
Jeremy was seven kilograms heavier than Gary.
Jeremy weighed 71 kilograms.

WHAT DO WE NEED TO FIND OUT?

Questioning:

What was Adrian's weight?

Planning and communicating a solution

Start at the end of the problem with Jeremy's weight which is 71 kilograms then work back through the sequence of factors to calculate Adrian's weight.

Begin by working out Gary's weight. Jeremy is seven kilograms heavier than Gary, so subtract the seven kilograms from Jeremy's weight of 71 kilograms.

$$71 \text{ kg} - 7 \text{ kg} = 64 \text{ kg.}$$

Gary is twice as heavy as Carter. Now that we know Gary's weight is 64 kilograms, we can calculate Carter's weight.

$$64 \div 2 = 32 \text{ kg.}$$

Carter's weight is 32 kilograms.

Adrian's weight is Carter's weight + 15 kilograms.

$$32 \text{ kg} + 15 \text{ kg} = 47 \text{ kg.}$$

Therefore, Adrian weighs 47 kilograms.

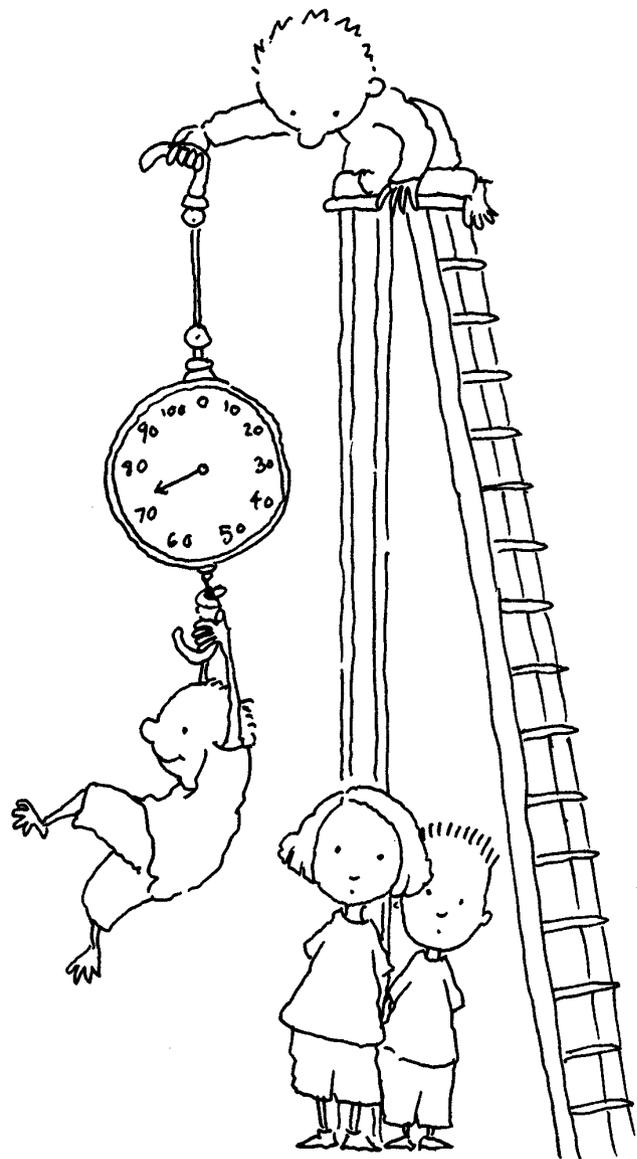
Reflecting and generalising

By using a step-by-step process, starting with the known and following a backwards sequence to find the missing information, the solution was easy to

calculate. An unmethodical approach could have resulted in the wrong operations being used or steps being missed out. Working backwards and using the opposite operations enabled a systematic approach, which can also be applied to any problem of a similar type.

Extension

To vary the problem, students could include more than four people in the problem, or add extra operations to the sequence.



Teaching Examples Working Backwards



EXAMPLE 3

In a spelling competition all the competitors were on stage together. After three minutes, a fifth of the students had made mistakes and were excluded from the competition. In the next five minutes half of those remaining were eliminated by extremely difficult words. Two minutes later four students were found cheating and were sent home. After fifteen minutes of the competition half of the remaining students had made mistakes and left the stage. In the last few minutes one more competitor made an unfortunate mistake and one contestant was left as the winner of the spelling competition. How many children originally entered the competition?

Understanding the problem

WHAT DO WE KNOW?

Students were participating in a spelling competition.

Students who made mistakes or cheated were eliminated.

There was one winner at the end of the competition

WHAT DO WE NEED TO FIND OUT?

Questioning:

How many children originally entered the competition?

Planning and communicating a solution

Start at the end and reverse the process. The winner

A few minutes before the end there was one more contestant = 2 spellers

Fifteen minutes into the competition, double the number = 4 spellers

Ten minutes into the competition add four to number = 8 spellers

Five minutes into the competition double the number = 16 spellers

Three minutes into the competition a fifth of the competitors had been eliminated so 16 spellers = $\frac{4}{5}$ of the total.

There were 20 children entered in the spelling competition.

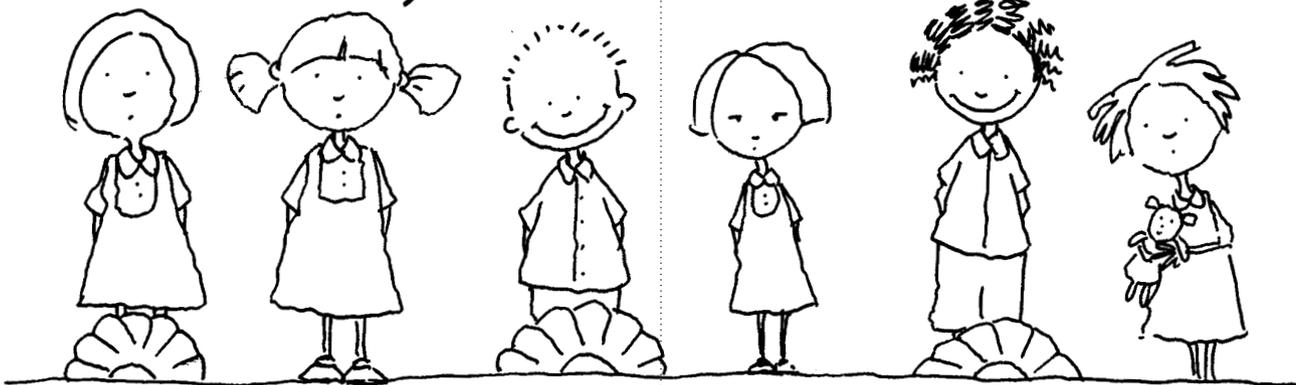
Reflecting and generalising

By reading the sentences in the problem carefully one at a time and recording all the known information it was possible to begin at the end and by reversing all operations to work backwards to reach a solution. Don't forget to work forward through the problem, once you have the solution, to check your answer.

Extension

The problem can be made more complicated by asking students to work with a variety of fractions or by including more steps.

e-l-i-m-i-n-a-t-e-d



BLM Working Backwards



★ Understanding the problem

What do you know? List the important facts from the problem. Draw a double line under your starting point.

.....

.....

★ What do you need to find out?

What is the problem asking you to do? What are you uncertain about? Do you understand all aspects of the problem? Is there any unfamiliar or unclear language?

.....



★ Planning and communicating a solution

Find your starting point. Work backwards in a logical step-by-step way. How many steps are required? Is all the information necessary? Will using objects to represent the people or places make the problem easier to solve?

★ Reflecting and generalising

Did the strategy work as planned? Is your answer correct? You can check this by working forward through the problem. Will you be able to apply this method of problem solving to other similar problems? Could you have used a different method to solve the problem?

.....

.....

★ Extension

How can this strategy be applied to more complicated problems involving bigger numbers and additional factors?

.....

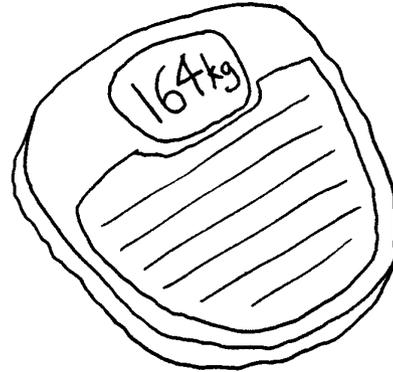
PROBLEM SOLVING TASK CARDS -Working Backwards

Problem 1

Measurement

Level
1

When three girls jumped on a weighing scale together, it read 164 kilograms. One girl stepped off and the scale moved down to 104 kilograms. One more girl jumped off and the scale showed 55 kilograms. What was each girl's weight?



Problem 2

Number 123

Level
1

Arnold baked cupcakes over the weekend. Each day during the week he took three cakes to school to share with his friends. On Saturday when he counted there were 18 left. How many had he baked?

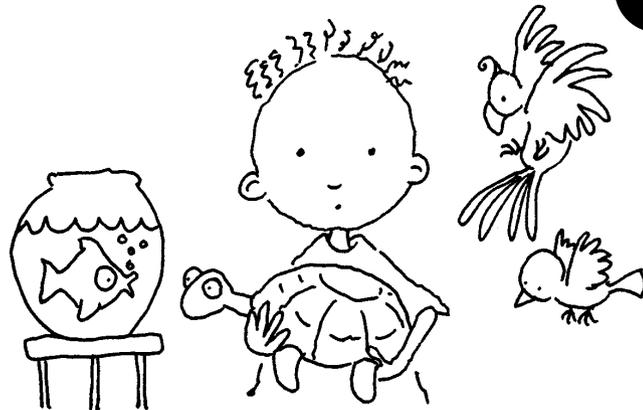


Problem 3

Number 123

Level
1

Daniel has lots of pets. He has four more goldfish than he has turtles. He has one less canary than goldfish. Six of his pets are birds (canaries and parrots). He has two parrots. How many pets does Daniel have?



PROBLEM SOLVING TASK CARDS -Working Backwards

Problem 4

Number **123**

Level
1

Jemima has twice as much money as Matthew. Jemima has four times as much money as Sally. Sally has \$3 more than Andrew. If Matthew has \$14, how much money do Andrew, Sally and Jemima have?



Problem 5

Number **123**

Level
1

Jack, Terence, Sharon and Alex are neighbours. Jack is half as old as Sharon. Sharon is three years older than Alex. Alex's and Sharon's ages added together equal 17 years. Terence is eight. Who is the youngest?



Problem 6

Number **123**

Level
1

A teacher bought five flags of different countries, to use in a class activity. She added them to the flags she already had in the classroom. She borrowed four more flags, but two of these weren't used. In the end ten flags were used in the activity. How many flags were there in the classroom already?



PROBLEM SOLVING TASK CARDS -Working Backwards

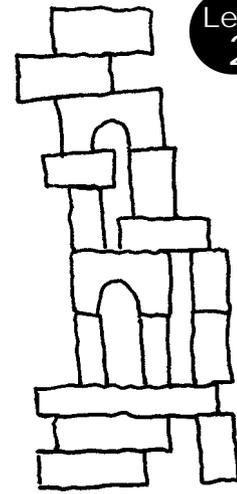
Problem 7

Space

Level
2

Six people entered a block building contest. Lynda built her pile of blocks twice as high as Selwyn's. Michelle created a pile that was three times higher than Lynda's pile. Warren built his pile one block higher than Michelle's. Jane's pile of blocks was six higher than Warren's.

If Adrian piled up 27 blocks, which was two blocks higher than Jane's pile, how high was each person's pile? How many blocks would be needed altogether?

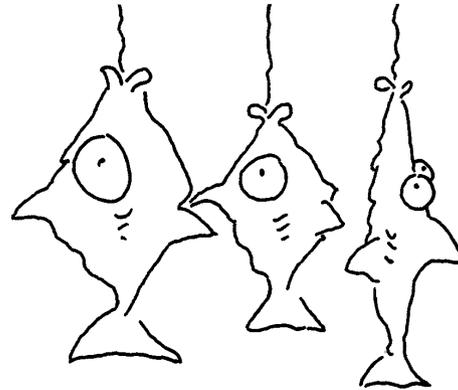


Problem 8

Measurement

Level
2

Four girls each caught a fish while at the beach. Teri's was double the size of Jane's. Jane's fish was shorter than Lynn's by nine centimetres. Lynn's was 18 centimetres longer than Reina's who caught a fish 30 centimetres long. How long were Teri's, Jane's and Lynn's fish?

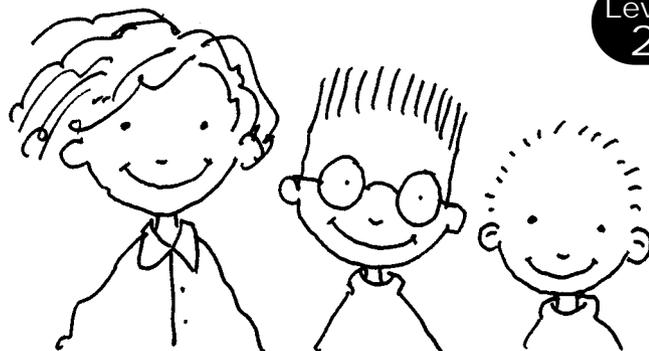


Problem 9

Number 123

Level
2

Joshua is five years older than David. Simon is four years older than Joshua and nine years older than David. Simon's and David's ages added together equal 15. How old are the boys?

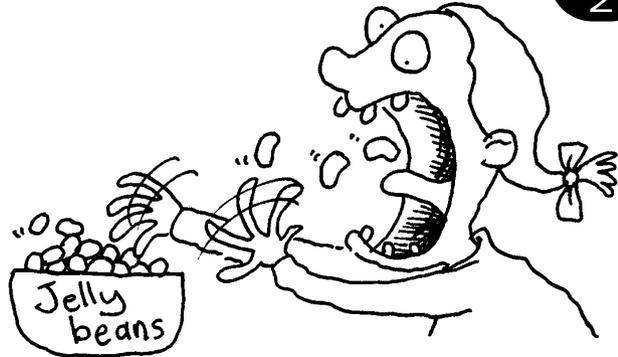


PROBLEM SOLVING TASK CARDS -Working Backwards

Problem 10 Number **123**

Level
2

At the party Jade ate less than four jelly beans. Nicole ate twice as many jelly beans as Jade. Kahlee had twice as many as Nicole. Chris had two more than Kahlee. Chris ate ten jelly beans. How many jelly beans were eaten at the party?



Problem 11 Number **123**

Level
2

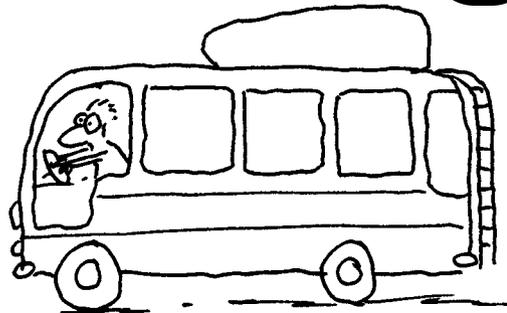
Mark, Neil, Frances and Patrick entered a skipping competition. Patrick skipped eight more times than Mark before his foot caught on the rope. Mark jumped three more times than Neal. Neal skipped half as many times as Frances. Frances skipped eighty times. How many times did Patrick skip?



Problem 12 Number **123**

Level
2

When Ariella climbs aboard there are already some people sitting the bus. At the next bus stop an additional five people get on and two people get off. Two stops later seven people climb on board. All 15 people get off the bus at the ferry. How many were on the bus when Ariella climbed on?



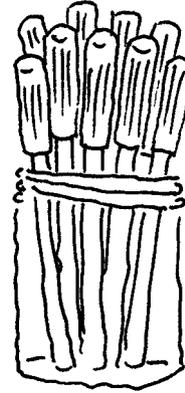
PROBLEM SOLVING TASK CARDS -Working Backwards

Problem 13

Number **123**

Level
2

There is an old jar packed full of textas on the table. There are twice as many red textas as blue and one more yellow than red. Eight textas are either yellow or green. There are three green textas. How many blue textas are in the jar?



Problem 14

Measurement
| | | | | | | | | |

Level
3

My new shoes arrived in a rectangular cardboard box. The length of the box was double the width and the width was double the height. The length was 28 centimetres. What was the volume of the shoebox?

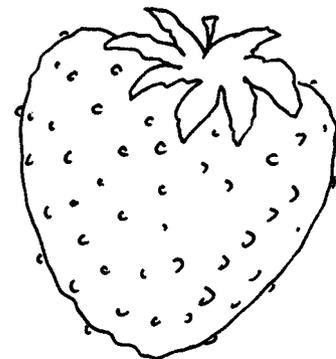


Problem 15

Number **123**

Level
3

Three people went strawberry collecting and picked 65 strawberries between them. At the first plant they each picked the same amount of strawberries. At the second plant they each collected three times the amount that they had collected at the first plant. After picking from the third plant they had five times the amount they had after picking strawberries from the first two plants. At the fourth plant they collected only five strawberries altogether. How many strawberries did each person collect at the first bush?

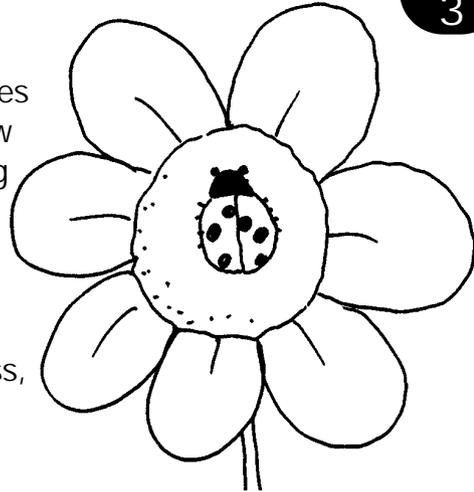


PROBLEM SOLVING TASK CARDS -Working Backwards

Problem 16 Number **123**

Level
3

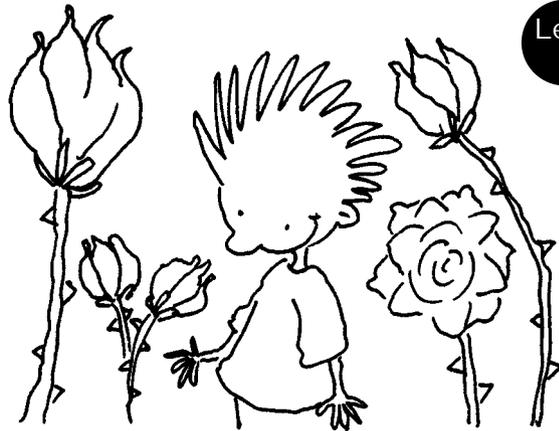
Twenty-four ladybirds were sitting at various places around the garden. One sixth of the ladybirds flew away to settle in the garden. Half of the remaining ladybirds sat on a yellow sunflower. Then half of those on the sunflower flew onto a fence post. There were no ladybirds on the hedge, but one fifth of the ladybirds on the sunflower flew onto the lavender bush. If one ladybird was on the grass, how many were on the tree trunk?



Problem 17 Number **123**

Level
3

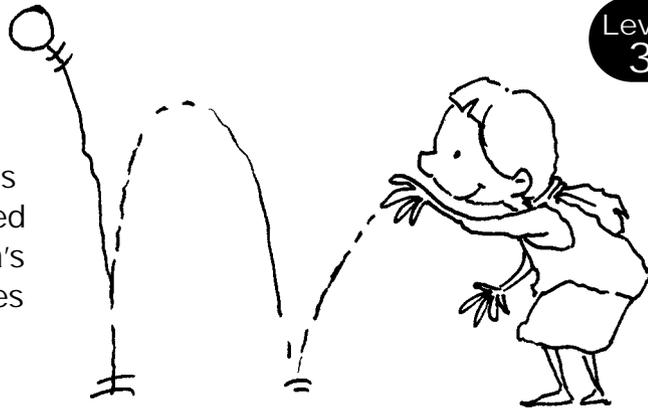
Heather loves roses. In her rose garden she has half as many pink roses as red, and four times as many red ones as white. There are 36 roses that are either yellow or white. Twenty are yellow. How many roses are in Heather's garden?



Problem 18 Number **123**

Level
3

Four children were bouncing balls. Jeff's ball bounced eight more times than Michael's. Michael's ball bounced half as many times as Olivia's. Olivia's bounced eight times. How many times did Jeff's ball bounce?



Answers to Task Cards

Problem 1

The weights of the three girls –

$$3 \text{ girls} = 164 \text{ kg}$$

$$2 \text{ girls} = 104 \text{ kg}$$

$$1 \text{ girl} = 55 \text{ kg}$$

$$2\text{nd girl}, 164 - 104 = 60 \text{ kg}$$

$$3\text{rd girl}, 104 - 55 = 49 \text{ kg}$$

Problem 2

Number of cupcakes baked by Arnold?

Monday – 3

Tuesday – 3

Wednesday – 3

Thursday – 3

Friday – 3

Saturday 18 cupcakes left

Therefore, working backwards,

$$18 + (5 \times 3) = 33 \text{ cupcakes.}$$

Problem 3

$$6 \text{ birds} - 2 \text{ parrots} = 4 \text{ canaries}$$

To find the number of goldfish, add 1 to the number of canaries.

$$4 + 1 = 5 \text{ goldfish}$$

To find the number of turtles, subtract 4 from the number of goldfish.

$$5 - 4 = 1 \text{ turtle}$$

Therefore, 2 parrots + 4 canaries + 5 goldfish + 1 turtle = 12 pets in total.

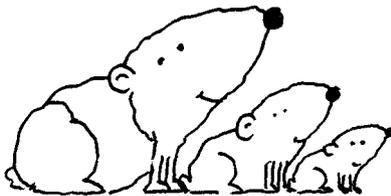
Problem 4

Matthew \$14

$$\text{Jemima } \$14 \times 2 = \$28$$

$$\text{Sally } \$28 \div 4 = \$7$$

$$\text{Andrew } \$7 - 3 = \$4$$



Problem 5

Terence is 8 years old.

Sharon's and Alex's ages total 17 years.

Sharon is 3 years older than Alex.

$$\text{So, } 17 - 3 = 14. 14 \div 2 = 7.$$

Alex is 7.

$$\text{Sharon is } 7 + 3 = 10.$$

$$\text{Jack is half Sharon's age so, } 10 \div 2 = 5.$$

Therefore, Jack is the youngest.

Problem 6

Original number of flags in classroom

$$+ 5 \text{ flags}$$

$$+ (4 - 2) = 2 \text{ flags}$$

$$= 10 \text{ flags}$$

Therefore working backwards,

$$10 - 2 - 5 = 3 \text{ flags in the classroom.}$$

Problem 7

Working backwards,

$$\text{Jane } 27 - 2 = 25$$

$$\text{Warren } 25 - 6 = 19$$

$$\text{Michelle } 19 - 1 = 18$$

$$\text{Lynda } 18 \div 3 = 6$$

$$\text{Selwyn } 6 \div 2 = 3$$

Total number of blocks

$$= 3 + 6 + 18 + 19 + 25 + 27 = 98$$

Problem 8

Reina's fish 30 cm

$$\text{Lynn's fish } 30 + 18 = 48 \text{ cm}$$

$$\text{Jane's fish } 48 - 9 = 39 \text{ cm}$$

$$\text{Teri's fish } 39 \times 2 = 78 \text{ cm}$$

Problem 9

David's and Simon's ages total 15.

Simon is 9 years older than David.

$$\text{So } 15 - 9 = 6. 6 \div 2 = 3.$$

David's age is 3.

$$\text{Simon's is } 3 + 9 = 12.$$

Joshua is 5 years older than David, so $3 + 5 = 8$.

Problem 10

Working backwards,

Chris, 10

Kahlee, $10 - 2 = 8$

Nicole, $8 \div 2 = 4$

Jane, $4 \div 2 = 2$

Total jelly beans eaten = $2 + 4 + 8 + 10 = 24$.

Problem 11

Frances, 80 skips

Neil, $80 \div 2 = 40$

Mark, $40 + 3 = 43$

Patrick, $43 + 8 = 51$

Problem 12

People on bus at beginning ?

Ariella gets on, + 1

1st stop, + 5 - 2

2nd stop, + 7

Total getting off bus, 15

Working backwards,

$15 - 7 = 8$

$8 + 2 - 5 = 5$

$5 - 1 = 4$

Four people were on the bus in the beginning.

Problem 13

Green textas, 3

Yellow and green = 8

$8 - 3 = 5$ yellow textas

One more yellow than red, $5 - 1 = 4$ red

Twice as many red as blue, $4 \div 2 = 2$ blue textas.

Problem 14

Length 28 cm

Length is double the width so,

$28 \div 2 = 14$ cm

Width is double the height so,

$14 \div 2 = 7$ cm

The volume is length x width x height,

$28 \times 14 \times 7 = 2,744$ cm².

Problem 15

Total strawberries 65

Working backwards,

$65 - 5 = 60$

$60 \div 5 = 12$

12 equals the total number of strawberries picked off the first and second plants. To work out how many were picked from the first bush you must divide by four to get four equal parts, one part being what was picked off the first plant and three parts being what was picked off the second plant. Therefore $12 \div 4 = 3$. So there were three strawberries picked off the first plant, one by each of the three people.

Problem 16

Total ladybirds minus $\frac{1}{6}$

$24 \div 6 = 4$

$24 - 4 = 20$ ladybirds left in garden

Half on sunflower $20 \div 2 = 10$

Half flew away onto fence post $10 \div 2 = 5$

0 on hedge

$\frac{1}{5}$ fly onto lavender bush $5 \div 5 = 1$

1 on grass

Total accounted for, $4 + 10 + 5 + 1 + 1 = 21$

So $24 - 21 = 3$ on tree trunk.

Problem 17

20 roses are yellow

36 are yellow or white

$36 - 20 = 16$ white

4 times as many red as white, $16 \times 4 = 64$ red

half as many pink as red $64 \div 2 = 32$ pink

$32 + 64 + 16 + 20 = 132$ roses

Problem 18

Olivia's ball bounced 8 times.

Michael's ball bounced half as many times as

Olivia's, so $8 \div 2 = 4$.

Jeff's ball bounced 8 more times than Michael's,

so $4 + 8 = 12$.